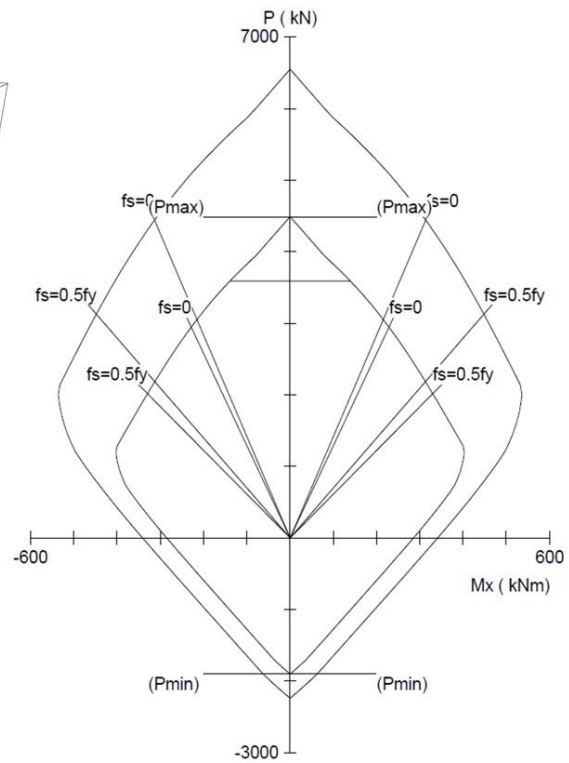
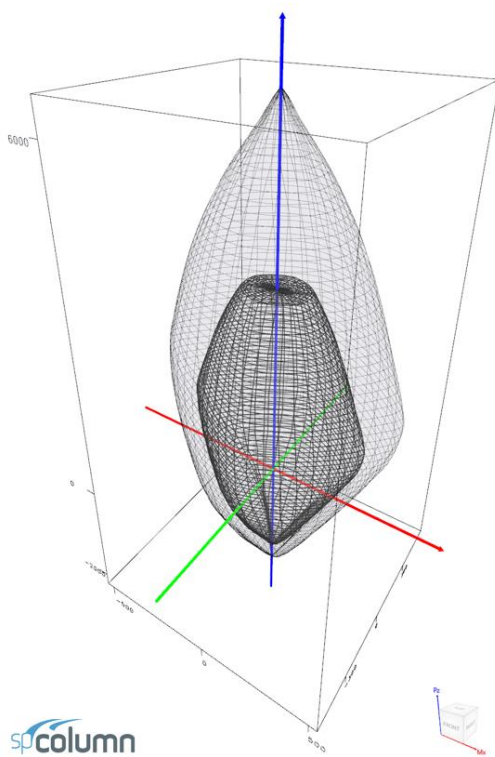
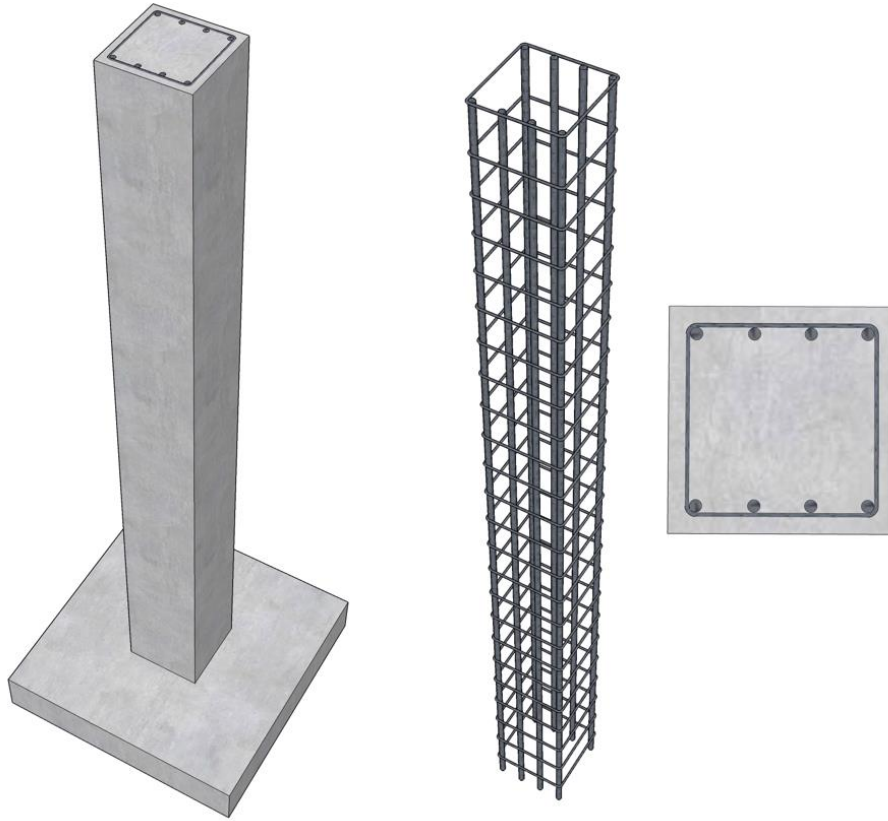


**Interaction Diagram - Tied Reinforced Concrete Column (Using CSA A23.3-94)**



### Interaction Diagram - Tied Reinforced Concrete Column

Develop an interaction diagram for the square tied concrete column shown in the figure below about the x-axis using CSA A23.3-14 provisions. Determine six control points on the interaction diagram and compare the calculated values in the Reference and with exact values from the complete interaction diagram generated by [spColumn](#) engineering software program from [StructurePoint](#).

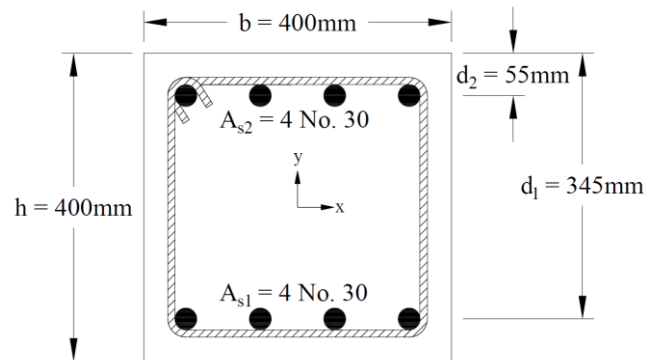


Figure 1 – Reinforced Concrete Column Cross-Section

## Contents

1. Pure Compression .....	3
1.1. Nominal axial resistance at zero eccentricity .....	3
1.2. Factored axial load resistance at zero eccentricity .....	3
1.3. Maximum factored axial load resistance .....	3
2. Bar Stress Near Tension Face of Member Equal to Zero, ( $\epsilon_s = f_s = 0$ ) .....	4
2.1. $c$ , $a$ , and strains in the reinforcement .....	4
2.2. Forces in the concrete and steel.....	4
2.3. $P_r$ and $M_r$ .....	5
3. Bar Stress Near Tension Face of Member Equal to $0.5 f_y$ , ( $f_s = -0.5 f_y$ ).....	6
3.1. $c$ , $a$ , and strains in the reinforcement .....	6
3.2. Forces in the concrete and steel.....	6
3.3. $P_r$ and $M_r$ .....	7
4. Bar Stress Near Tension Face of Member Equal to $f_y$ , ( $f_s = -f_y$ ).....	8
4.1. $c$ , $a$ , and strains in the reinforcement .....	8
4.2. Forces in the concrete and steel.....	9
4.3. $P_r$ and $M_r$ .....	9
5. Pure Bending.....	10
5.1. $c$ , $a$ , and strains in the reinforcement .....	10
5.2. Forces in the concrete and steel.....	10
5.3. $P_r$ and $M_r$ .....	11
6. Pure Tension.....	12
6.1. Strength under pure axial tension ( $P_r$ ).....	12
6.2. Corresponding Moment ( $M_r$ ).....	12
7. Column Interaction Diagram - spColumn Software.....	13
8. Summary and Comparison of Design Results.....	21
9. Conclusions & Observations.....	22

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## Code

Design of Concrete Structures (CSA A23.3-94)

## Reference

Reinforced Concrete Mechanics and Design, 1<sup>st</sup> Canadian Edition, 2000, James MacGregor and Fred Michael  
Bratlett, Prentice Hall Canada Inc.

## Design Data

$$f_c' = 35 \text{ MPa}$$

$$f_y = 400 \text{ MPa}$$

Cover = 55 mm to the center of the reinforcement

Column 400 mm x 400 mm

Top reinforcement = 4 No. 30

Bottom reinforcement = 4 No. 30

## Solution

Use the traditional hand calculations approach to generate the interaction diagram for the concrete column section shown above by determining the following six control points:

Point 1: Pure compression

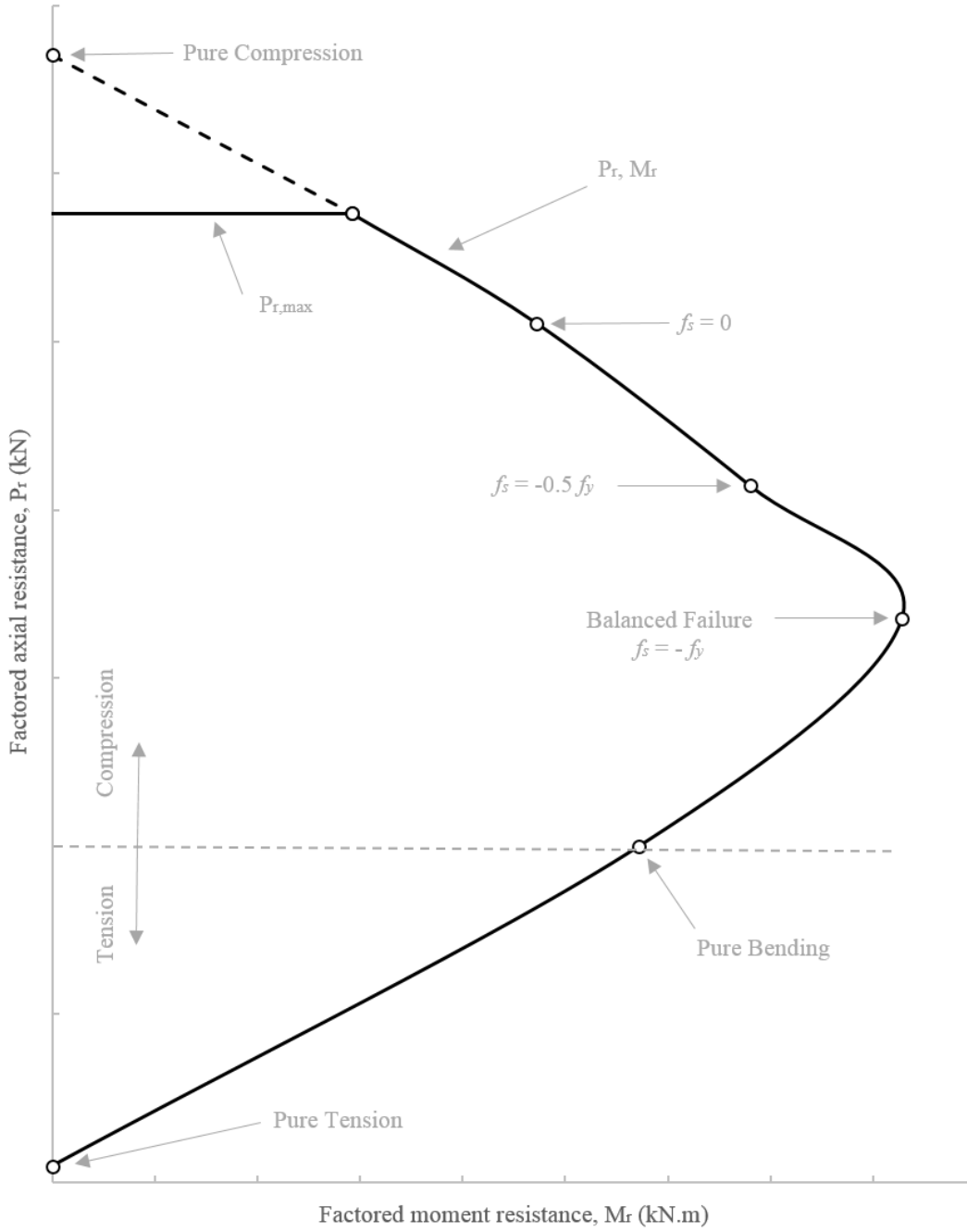
Point 2: Bar stress near tension face of member equal to zero, ( $f_s = 0$ )

Point 3: Bar stress near tension face of member equal to  $0.5 f_y$  ( $f_s = -0.5 f_y$ )

Point 4: Bar stress near tension face of member equal to  $f_y$  ( $f_s = -f_y$ )

Point 5: Pure bending

Point 6: Pure tension



**Figure 2 – Control Points**

## 1. Pure Compression

### 1.1. Nominal axial resistance at zero eccentricity

$$P_o = \alpha_1 f'_c (A_g - A_{st}) + f_y A_{st}$$

$$P_o = 0.798 \times 35 \times (400 \times 400 - 8 \times 700) + 400 \times 8 \times 700 = 6550 \text{ kN}$$

$$\text{Where } \alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67$$

CSA A23.3-94 (Equation 10-1)

$$\alpha_1 = 0.85 - 0.0015 \times 35 = 0.798 \geq 0.67$$

### 1.2. Factored axial load resistance at zero eccentricity

Since this column is a tied column with steel strain in compression:

$$P_{ro} = \alpha_1 \phi_c f'_c (A_g - A_{st}) + \phi_s f_y A_{st}$$

CSA A23.3-94 (Equation 10-10)

$$P_{ro} = 0.798 \times 0.6 \times 35 \times (400 \times 400 - 8 \times 700) + 0.85 \times 400 \times 8 \times 700 = 4490 \text{ kN}$$

Where:

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

### 1.3. Maximum factored axial load resistance

$$P_{r,max} = 0.80 P_{ro}$$

CSA A23.3-94 (Equation 10-9)

$$P_{r,max} = (0.2 + 0.002 \times 400) \times 4490 = 4490 \text{ kN} \leq 0.80 P_{ro} = 0.80 \times 4490 = 3592 \text{ kN}$$

$$P_{r,max} = 3592 \text{ kN}$$

**2. Bar Stress Near Tension Face of Member Equal to Zero, ( $\epsilon_s = f_s = 0$ )**

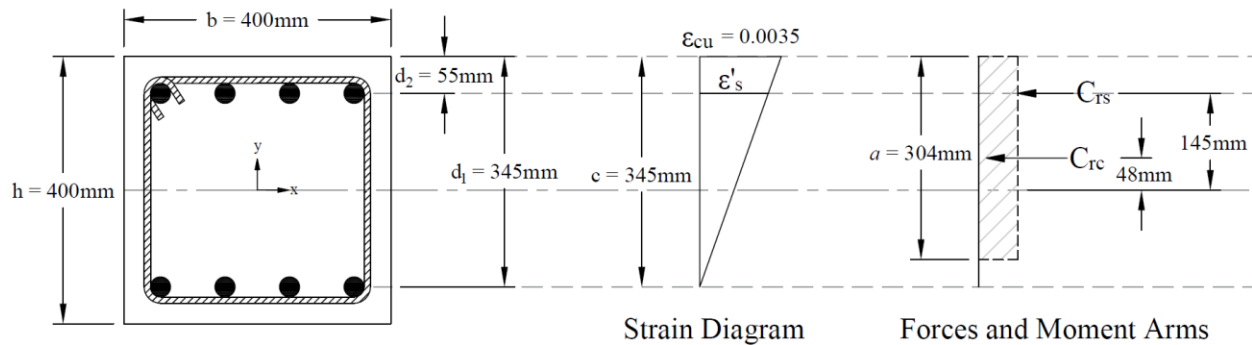


Figure 3 – Strains, Forces, and Moment Arms ( $\epsilon_t = f_s = 0$ )

Strain  $\epsilon_s$  is zero in the extreme layer of tension steel. This case is considered when calculating an interaction diagram because it marks the change from compression lap splices being allowed on all longitudinal bars, to the more severe requirement of tensile lap splices. CSA A23.3-94 (12.15 and 16)

**2.1. c, a, and strains in the reinforcement**

$$c = d_1 = 345 \text{ mm}$$

Where  $c$  is distance from extreme compression fiber to neutral axis.

CSA A23.3-94 (10.0)

$$a = \beta_1 \times c = 0.883 \times 345 = 304 \text{ mm}$$

CSA A23.3-94 (10.1.7)

Where:

$a$  = Depth of equivalent rectangular stress block

CSA A23.3-94 (10.0)

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

CSA A23.3-94 (Equation 10-2)

$$\epsilon_s = 0$$

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

$$\epsilon_{cu} = 0.0035$$

CSA A23.3-94 (10.1.3)

$$\epsilon'_s = (c - d_2) \times \frac{\epsilon_{cu}}{c} = (345 - 55) \times \frac{0.0035}{345} = 0.00294 \text{ (Compression)} > \epsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

**2.2. Forces in the concrete and steel**

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 304 \times 400 = 2040 \text{ kN}$$

CSA A23.3-94 (10.1.7)

$$f_s = 0 \text{ kN} \rightarrow T_{rs} = \phi_s \times f_s \times A_{s1} = 0 \text{ kN}$$

Since  $\varepsilon_s' > \varepsilon_y \rightarrow$  compression reinforcement has yielded

$$\therefore f_s' = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $\alpha_1 \phi_c f_c'$  from  $\phi_s f_s'$  before computing  $C_{rs}$ :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

### 2.3. $P_r$ and $M_r$

$$P_r = C_{rc} + C_{rs} - T_{rs} = 2040 + 905 - 0 = 2945 \text{ kN}$$

$$M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_2 \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right)$$

$$M_r = 2040 \times \left( \frac{400}{2} - \frac{304}{2} \right) + 905 \times \left( \frac{400}{2} - 55 \right) + 0 \times \left( 345 - \frac{400}{2} \right) = 229 \text{ kN.m}$$



**3. Bar Stress Near Tension Face of Member Equal to  $0.5 f_y$ , ( $f_s = -0.5 f_y$ )**

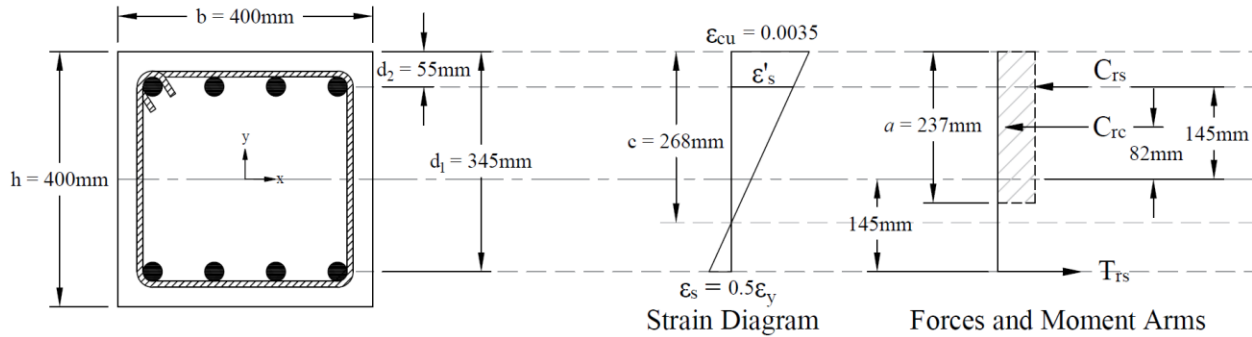


Figure 4 – Strains, Forces, and Moment Arms ( $f_s = -0.5 f_y$ )

**3.1. c, a, and strains in the reinforcement**

$$\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = \frac{\varepsilon_y}{2} = \frac{0.002}{2} = 0.001 < \varepsilon_y \rightarrow \text{tension reinforcement has not yielded}$$

$$\phi_c = 0.6$$

CSA A23.3-94 (8.4.2)

$$\phi_s = 0.85$$

CSA A23.3-94 (8.4.3)

$$\varepsilon_{cu} = 0.0035$$

CSA A23.3-94 (10.1.3)

$$c = \frac{d_1}{\varepsilon_s + \varepsilon_{cu}} \times \varepsilon_{cu} = \frac{345}{0.001 + 0.0035} \times 0.0035 = 268 \text{ mm}$$

Where  $c$  is depth of the neutral axis measured from the compression edge of the column section.

CSA A23.3-94 (10.0)

$$a = \beta_1 \times c = 0.883 \times 268 = 237 \text{ mm}$$

CSA A23.3-94 (10.1.7)

Where:

$a$  = Depth of equivalent rectangular stress block

CSA A23.3-94 (10.0)

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

CSA A23.3-94 (Equation 10-2)

$$\varepsilon'_s = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (268 - 55) \times \frac{0.0035}{268} = 0.00278 \text{ (Compression)} > \varepsilon_y$$

**3.2. Forces in the concrete and steel**

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 237 \times 400 = 1586 \text{ kN}$$

CSA A23.3-94 (10.1.7)

$$f_s = \varepsilon_s \times E_s = 0.001 \times 200,000 = 200 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 200 \times 2800 = 476 \text{ kN}$$

Since  $\varepsilon_s' > \varepsilon_y \rightarrow$  compression reinforcement has yielded

$$\therefore f_s' = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $\alpha_1 \phi_c f_c'$  from  $\phi_s f_s'$  before computing  $C_{rs}$ :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

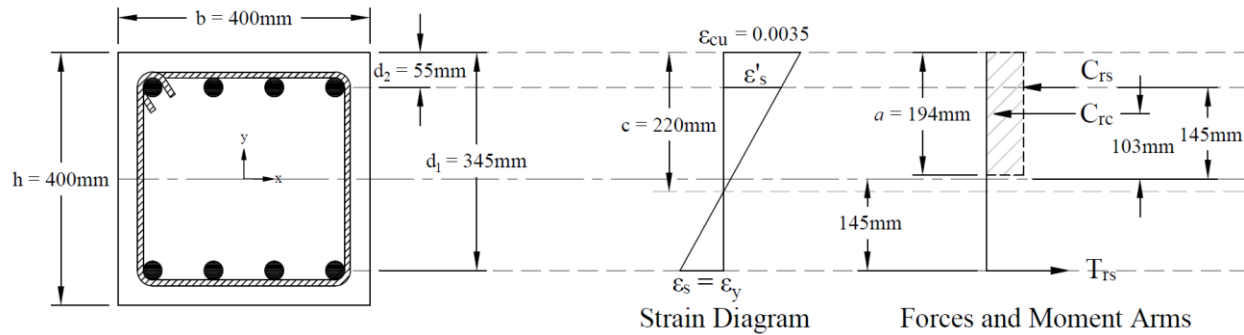
### 3.3. $P_r$ and $M_r$

$$P_r = C_{rc} + C_{rs} - T_{rs} = 1586 + 905 - 476 = 2015 \text{ kN}$$

$$M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_2 \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right)$$

$$M_r = 1586 \times \left( \frac{400}{2} - \frac{237}{2} \right) + 905 \times \left( \frac{400}{2} - 55 \right) + 476 \times \left( 345 - \frac{400}{2} \right) = 330 \text{ kN.m}$$

**4. Bar Stress Near Tension Face of Member Equal to  $f_y$ , ( $f_s = -f_y$ )**



**Figure 5 – Strains, Forces, and Moment Arms ( $f_s = -f_y$ )**

This strain distribution is called the balanced failure case and the compression-controlled strain limit. It marks the change from compression failures originating by crushing of the compression surface of the section, to tension failures initiated by yield of longitudinal reinforcement.

**4.1.  $c$ ,  $a$ , and strains in the reinforcement**

$$\epsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\epsilon_s = \epsilon_y = 0.002 \rightarrow \text{tension reinforcement has yielded}$$

$$\phi_c = 0.6 \quad \text{CSA A23.3-94 (8.4.2)}$$

$$\phi_s = 0.85 \quad \text{CSA A23.3-94 (8.4.3)}$$

$$\epsilon_{cu} = 0.0035 \quad \text{CSA A23.3-94 (10.1.3)}$$

$$c = \frac{d_1}{\epsilon_s + \epsilon_{cu}} \times \epsilon_{cu} = \frac{345}{0.002 + 0.0035} \times 0.0035 = 220 \text{ mm}$$

Where  $c$  is depth of the neutral axis measured from the compression edge of the column section.

$$\text{CSA A23.3-94 (10.0)}$$

$$a = \beta_1 \times c = 0.883 \times 220 = 194 \text{ mm} \quad \text{CSA A23.3-94 (10.1.7)}$$

Where:

$$a = \text{Depth of equivalent rectangular stress block} \quad \text{CSA A23.3-94 (10.0)}$$

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67 \quad \text{CSA A23.3-94 (Equation 10-2)}$$

$$\epsilon'_s = (c - d_2) \times \frac{\epsilon_{cu}}{c} = (220 - 55) \times \frac{0.0035}{220} = 0.00262 \text{ (Compression)} > \epsilon_y$$

4.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 194 \times 400 = 1298 \text{ kN} \quad \text{CSA A23.3-94 (10.1.7)}$$

$$f_s = f_y = 400 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN}$$

Since  $\epsilon'_s > \epsilon_y \rightarrow$  compression reinforcement has yielded

$$\therefore f'_s = f_y = 400 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $\alpha_1 \phi_c f'_c$  from  $\phi_s f'_s$  before computing  $C_{rs}$ :

$$C_{rs} = (\phi_s \times f'_s - \alpha_1 \times \phi_c \times f'_c) \times A_{s2} = (0.85 \times 400 - 0.798 \times 0.6 \times 35) \times 2800 = 905 \text{ kN}$$

4.3.  $P_r$  and  $M_r$

$$P_r = C_{rc} + C_{rs} - T_{rs} = 1406 + 901 - 952 = 1355 \text{ kN}$$

$$M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_2 \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right)$$

$$M_r = 1298 \times \left( \frac{400}{2} - \frac{194}{2} \right) + 905 \times \left( \frac{400}{2} - 55 \right) + 952 \times \left( 345 - \frac{400}{2} \right) = 403 \text{ kN.m}$$

## 5. Pure Bending

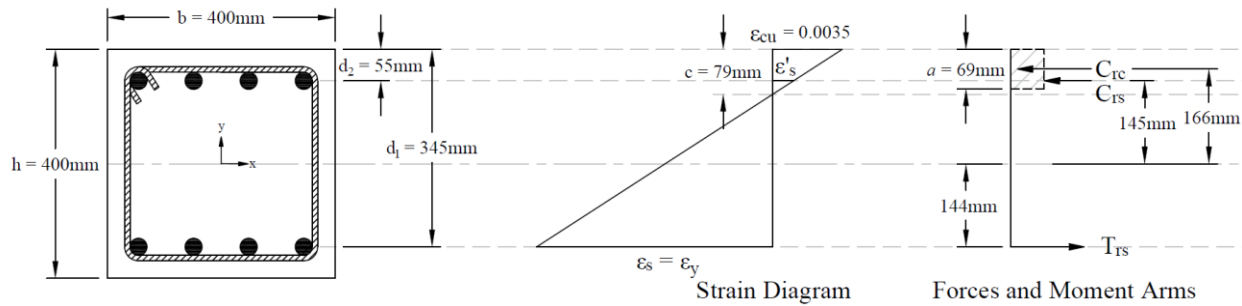


Figure 6 – Strains, Forces, and Moment Arms (Pure Moment)

This corresponds to the case where the factored axial load resistance,  $P_r$ , is equal to zero. Iterative procedure is used to determine the factored moment resistance as follows:

### 5.1. $c$ , $a$ , and strains in the reinforcement

Try  $c = 78.55$  mm

Where  $c$  is depth of the neutral axis measured from the compression edge of the column section.

$$\underline{CSA A23.3-94 (10.0)}$$

$$a = \beta_1 \times c = 0.883 \times 78.55 = 69 \text{ mm}$$

$$\underline{CSA A23.3-94 (10.1.7)}$$

Where:

$$\beta_1 = 0.97 - 0.0025 \times f'_c = 0.97 - 0.0025 \times 35 = 0.883 > 0.67$$

$$\underline{CSA A23.3-94 (Equation 10-2)}$$

$$\varepsilon_{cu} = 0.0035$$

$$\underline{CSA A23.3-94 (10.1.3)}$$

$$\varepsilon_y = \frac{F_y}{E_s} = \frac{400}{200,000} = 0.002$$

$$\varepsilon_s = (d_1 - c) \times \frac{\varepsilon_{cu}}{c} = (345 - 78.55) \times \frac{0.0035}{78.55} = 0.01187 \text{ (Tension)} > \varepsilon_y \rightarrow \text{tension reinforcement has yielded}$$

$$\phi_c = 0.6$$

$$\underline{CSA A23.3-94 (8.4.2)}$$

$$\phi_s = 0.85$$

$$\underline{CSA A23.3-94 (8.4.3)}$$

$$\varepsilon'_s = (c - d_2) \times \frac{\varepsilon_{cu}}{c} = (78.55 - 55) \times \frac{0.0035}{78.55} = 0.00105 \text{ (Compression)} < \varepsilon_y$$

### 5.2. Forces in the concrete and steel

$$C_{rc} = \alpha_1 \times \phi_c \times f'_c \times a \times b = 0.798 \times 0.6 \times 35 \times 69 \times 400 = 474 \text{ kN}$$

$$\underline{CSA A23.3-94 (10.1.7)}$$

$$f_s = f_y = 400 \text{ MPa}$$

$$T_{rs} = \phi_s \times f_s \times A_{s1} = 0.85 \times 400 \times 2800 = 952 \text{ kN}$$

Since  $\varepsilon_s' < \varepsilon_y \rightarrow$  compression reinforcement has not yielded

$$\therefore f_s' = \varepsilon_s' \times E_s = 0.00105 \times 200,000 = 210 \text{ MPa}$$

The area of the reinforcement in this layer has been included in the area ( $ab$ ) used to compute  $C_c$ . As a result, it is necessary to subtract  $\alpha_1 \phi_c f_c'$  from  $\phi_s f_s'$  before computing  $C_{rs}$ :

$$C_{rs} = (\phi_s \times f_s' - \alpha_1 \times \phi_c \times f_c') \times A_{s2} = (0.85 \times 210 - 0.798 \times 0.6 \times 35) \times 2800 = 477 \text{ kN}$$

### 5.3. $P_r$ and $M_r$

$$P_r = C_{rc} + C_{rs} - T_{rs} = 474 + 477 - 952 \approx 0 \text{ kN}$$

The assumption that  $c = 78.55 \text{ mm}$  is correct

$$M_r = C_{rc} \times \left( \frac{h}{2} - \frac{a}{2} \right) + C_{rs} \times \left( \frac{h}{2} - d_2 \right) + T_{rs} \times \left( d_1 - \frac{h}{2} \right)$$

$$M_r = 474 \times \left( \frac{400}{2} - \frac{69}{2} \right) + 477 \times \left( \frac{400}{2} - 55 \right) + 952 \times \left( 345 - \frac{400}{2} \right) = 285 \text{ kN.m}$$

## 6. Pure Tension

The final loading case to be considered is concentric axial tension. The strength under pure axial tension is computed by assuming that the section is completely cracked through and subjected to a uniform strain greater than or equal to the yield strain in tension. The strength under such a loading is equal to the yield strength of the reinforcement in tension.

### 6.1. Strength under pure axial tension ( $P_{rt}$ )

$$P_{rt} = \phi_s \times f_y \times (A_{s1} + A_{s2}) = 0.85 \times 400 \times (2800 + 2800) = 1904 \text{ kN}$$

### 6.2. Corresponding Moment ( $M_{rt}$ )

Since the section is symmetrical

$$M_{rt} = 0 \text{ kN.m}$$

## 7. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility. For this column section, we ran in investigation mode with control points using the CSA A23.3-94. In lieu of using program shortcuts, spSection (Figure 9) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

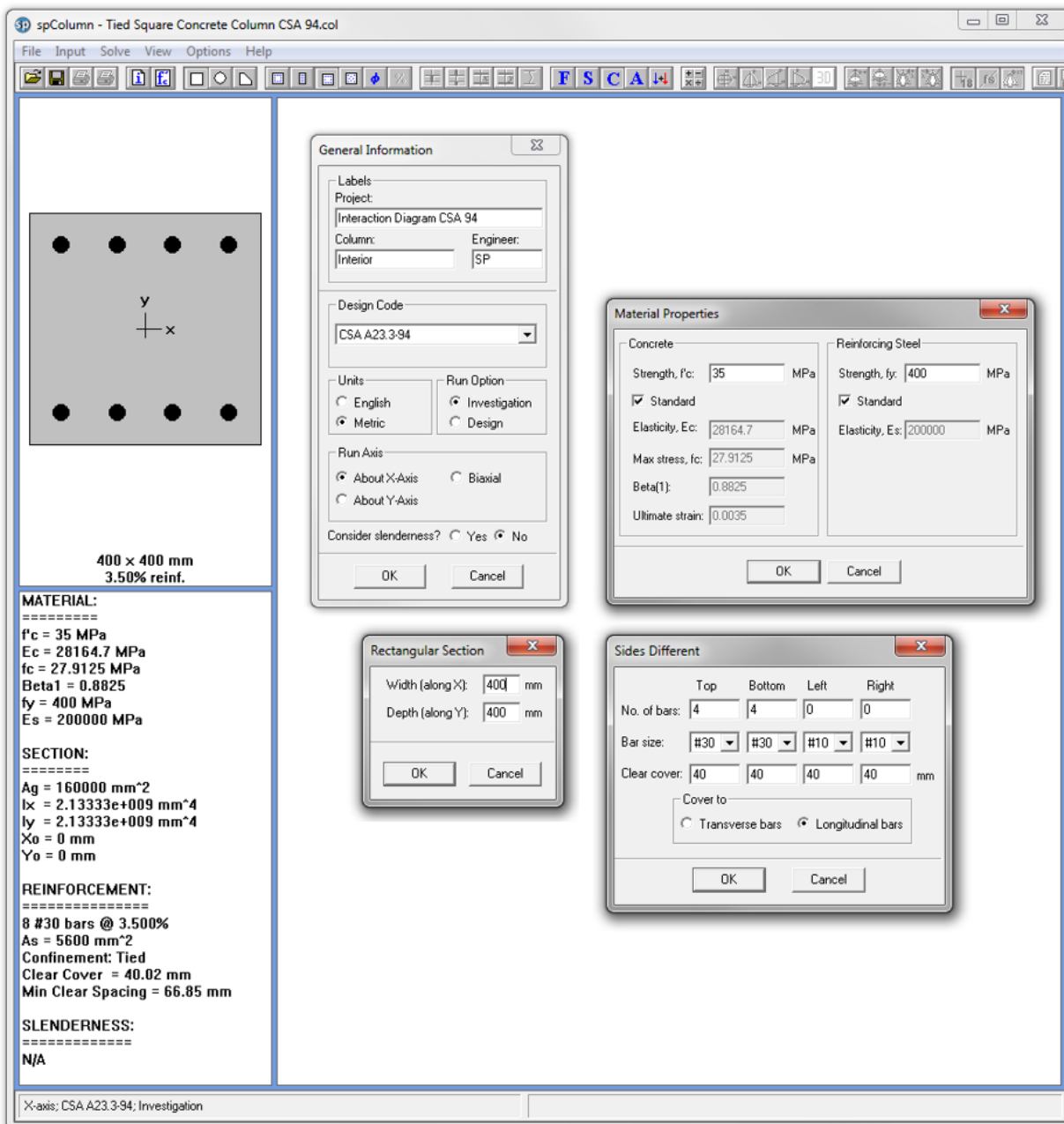


Figure 7 – Generating spColumn Model



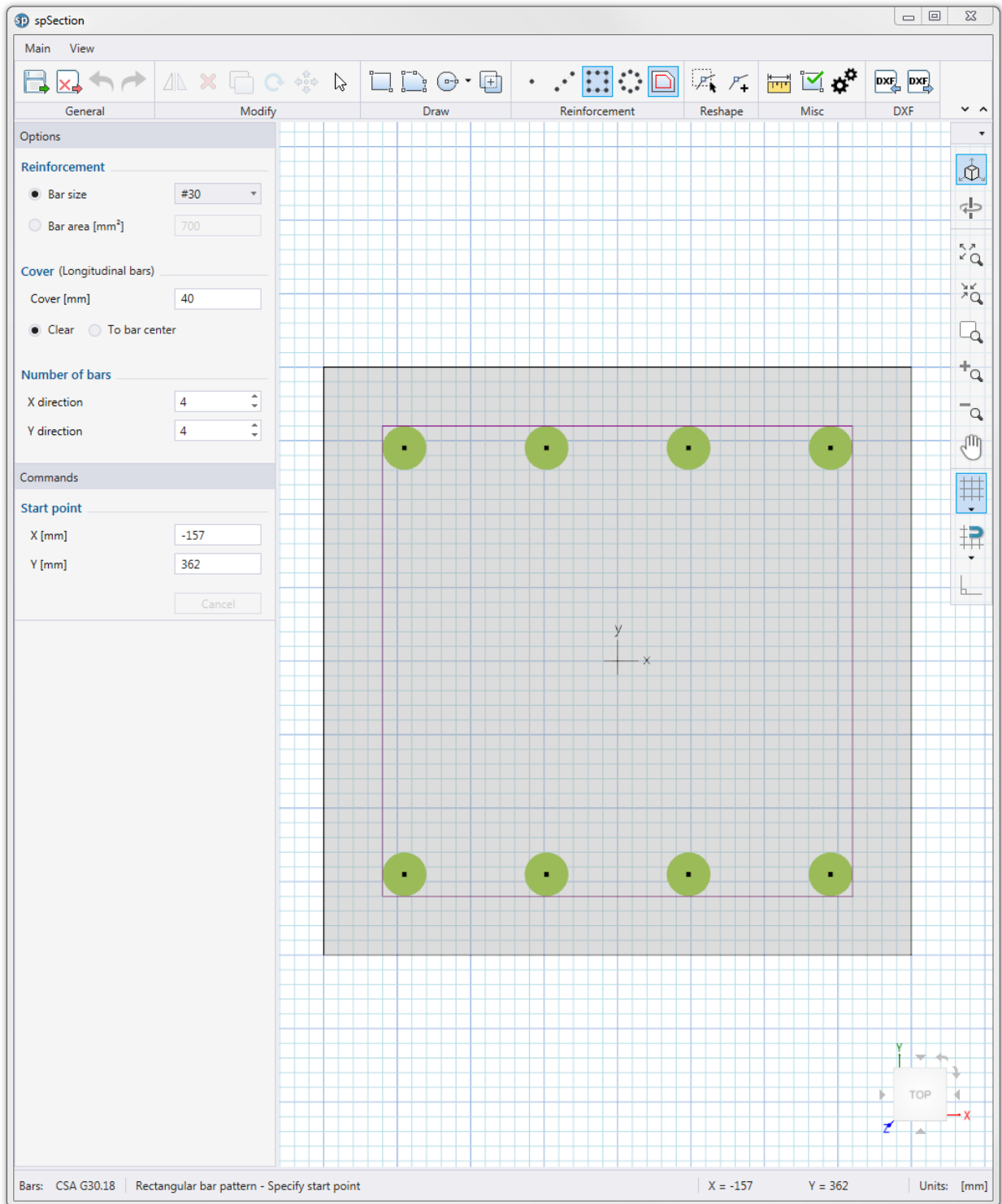


Figure 8 – spColumn Model Editor (spSection)

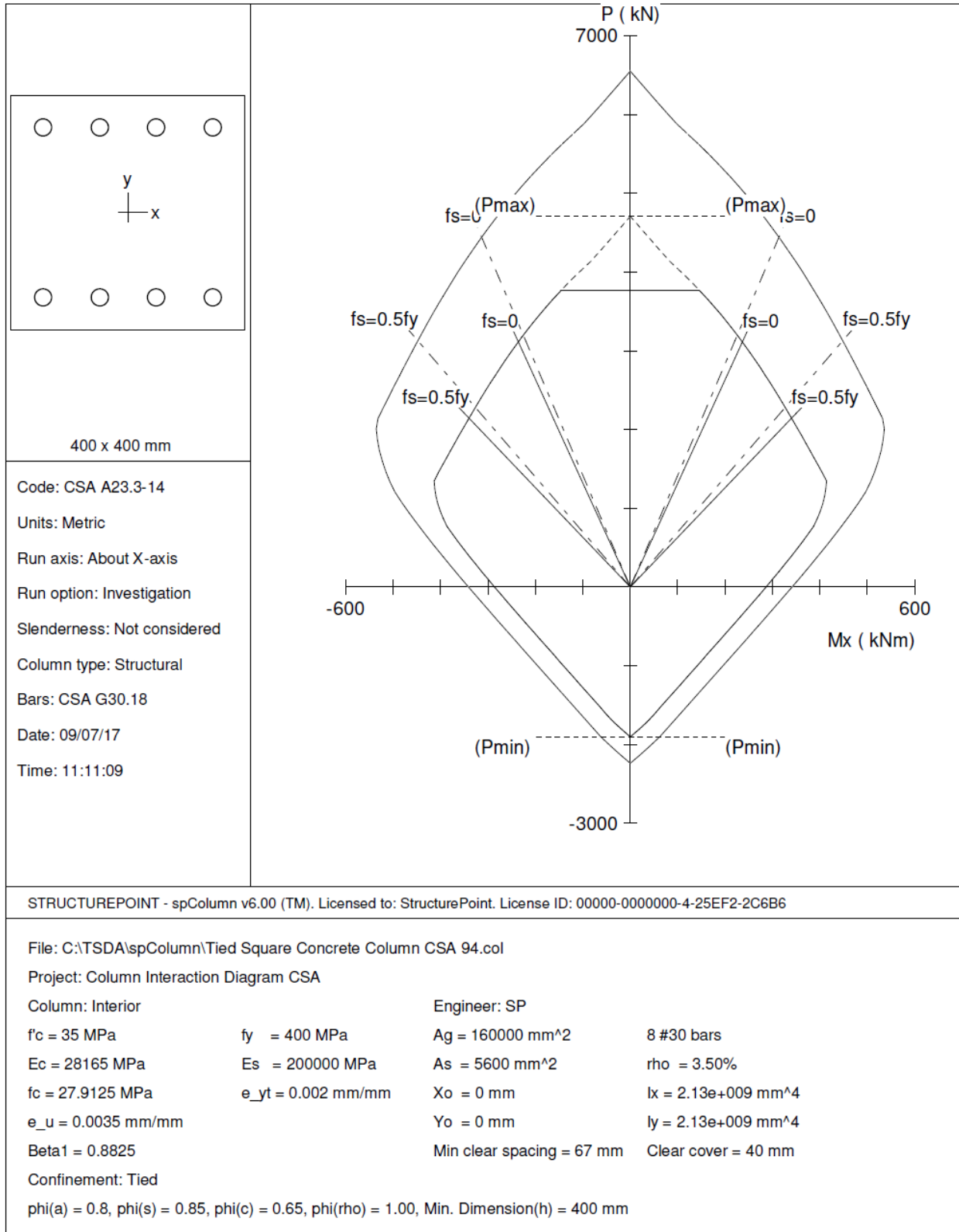


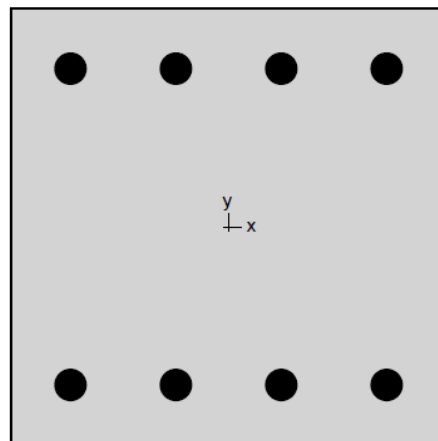
Figure 9 – Column Section Interaction Diagram about the X-Axis (spColumn)



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spColumn v6.00  
Computer program for the Strength Design of Reinforced Concrete Sections  
Copyright - 1988-2017, STRUCTUREPOINT, LLC.  
All rights reserved

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## Contents

1. General Information .....	3
2. Material Properties.....	3
2.1. Concrete.....	3
2.2. Steel .....	3
3. Section.....	3
3.1. Shape and Properties.....	3
3.2. Section Figure .....	4
4. Reinforcement .....	4
4.1. Bar Set: CSA G30.18 .....	4
4.2. Confinement and Factors .....	4
4.3. Arrangement.....	4
4.4. Bars Provided.....	5
5. Control Points .....	5

## List of Figures

Figure 1: Column section.....	4
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## 1. General Information

File Name	C:\TSDA...\Tied Square Concrete Column CSA 94.col
Project	Column Interaction Diagram CSA
Column	Interior
Engineer	SP
Code	CSA A23.3-14
Bar Set	CSA G30.18
Units	Metric
Run Option	Investigation
Run Axis	X - axis
Slenderness	Not Considered
Column Type	Structural

## 2. Material Properties

### 2.1. Concrete

Type	Standard
$f_c$	35 MPa
$E_c$	28164.7 MPa
$f_c$	27.9125 MPa
$\epsilon_u$	0.0035 mm/mm
$\beta_1$	0.8825

### 2.2. Steel

Type	Standard
$f_y$	400 MPa
$E_s$	200000 MPa
$\epsilon_{yt}$	0.002 mm/mm

## 3. Section

### 3.1. Shape and Properties

Type	Rectangular
Width	400 mm
Depth	400 mm
$A_g$	160000 mm <sup>2</sup>
$I_x$	2.13333e+009 mm <sup>4</sup>
$I_y$	2.13333e+009 mm <sup>4</sup>
$r_x$	115.47 mm
$r_y$	115.47 mm
$X_o$	0 mm
$Y_o$	0 mm

### 3.2. Section Figure

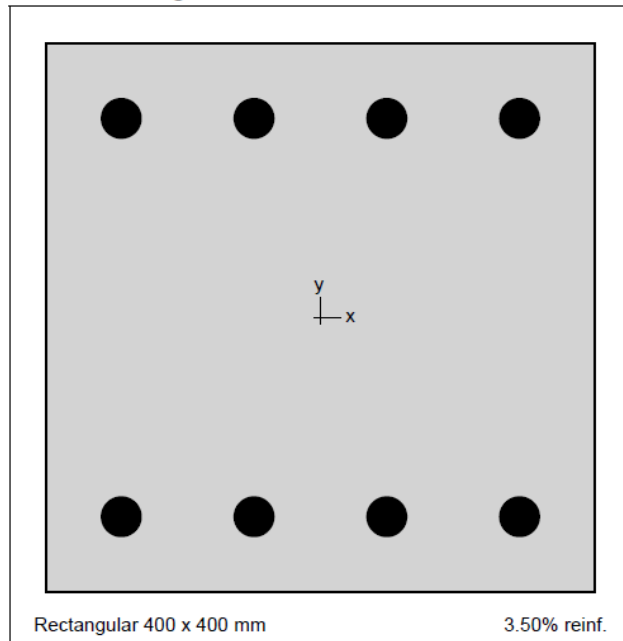


Figure 1: Column section

## 4. Reinforcement

### 4.1. Bar Set: CSA G30.18

Bar	Diameter mm	Area mm <sup>2</sup>	Bar	Diameter mm	Area mm <sup>2</sup>	Bar	Diameter mm	Area mm <sup>2</sup>
#10	11.30	100.00	#15	16.00	200.00	#20	19.50	300.00
#25	25.20	500.00	#30	29.90	700.00	#35	35.70	1000.00
#45	43.70	1500.00	#55	56.40	2500.00			

### 4.2. Confinement and Factors

Confinement type	Tied
For #55 bars or less	#10 ties
For larger bars	#15 ties
<b>Material Resistance Factors</b>	
Axial compression, (a)	0.8
Steel ( $\Phi_s$ )	0.85
Concrete ( $\Phi_c$ )	0.65
Minimum dimension, h	400 mm

### 4.3. Arrangement

Pattern	Sides different
Bar layout	---
Cover to	Longitudinal bars
Clear cover	---

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C:\TSDA\spColumn\Tied Square Concrete Column CSA 94.col

Page | 5  
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Bars	---
Total steel area, $A_s$	5600 mm <sup>2</sup>
rho	3.50 %
Minimum clear spacing	67 mm

#### 4.4. Bars Provided

	Bars	Cover mm
Top	4 #30	40
Bottom	4 #30	40
Left	0 #10	40
Right	0 #10	40

#### 5. Control Points

About Point	P kN	X-Moment kNm	Y-Moment kNm	NA Depth mm	$d_t$ Depth mm	$\epsilon_t$
X @ Max compression	4705.3	0.00	0.00	805	345	-0.00200
X @ Allowable comp.	3764.2	146.16	0.00	412	345	-0.00057
X @ $f_s = 0.0$	3111.1	236.23	0.00	345	345	0.00000
X @ $f_s = 0.5 f_y$	2144.0	339.98	0.00	268	345	0.00100
X @ Balanced point	1355.5	413.81	0.00	220	345	0.00200
X @ Pure bending	0.0	286.39	0.00	79	345	0.01188
X @ Max tension	-1904.0	0.00	0.00	0	345	9.99999
-X @ Max compression	4705.3	0.00	0.00	805	345	-0.00200
-X @ Allowable comp.	3764.2	-146.16	0.00	412	345	-0.00057
-X @ $f_s = 0.0$	3111.1	-236.23	0.00	345	345	0.00000
-X @ $f_s = 0.5 f_y$	2144.0	-339.98	0.00	268	345	0.00100
-X @ Balanced point	1355.5	-413.81	0.00	220	345	0.00200
-X @ Pure bending	0.0	-286.39	0.00	79	345	0.01188
-X @ Max tension	-1904.0	0.00	0.00	0	345	9.99999

## 8. Summary and Comparison of Design Results

Table 1 - Comparison of Results						
Support	$P_r$ , kN			$M_r$ , kN.m		
	Hand	Reference*	spColumn	Hand	Reference*	spColumn
Max compression	4490	4490	4490	0	0	0
Allowable compression	3592	3592	3592	---	---	---
$f_s = 0.0$	2945	2945	2945	229	229	229
$f_s = 0.5 f_y$	2015	2015	2016	330	330	330
Balanced point	1251	1253	1251	403	403	403
Pure bending	0	0	0	285	285	285
Max tension	1904	1904	1904	0	0	0

\* Reinforced Concrete Mechanic and Design, 1<sup>st</sup> Canadian Edition, James MacGregor and Fred Bartlett – Example 11-1

In all of the hand calculations and the reference used illustrated above, the results are in precise agreement with the automated exact results obtained from the [spColumn](#) program.



### 9. Conclusions & Observations

The analysis of the reinforced concrete section performed by [spColumn](#) conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility.

In the calculation shown above a P-M interaction diagram was generated with moments about the X-Axis (Uniaxial bending). Since the reinforcement in the section is not symmetrical, a different P-M interaction diagram is needed for the other orthogonal direction about the Y-Axis (See the following Figure for the case where  $f_s = f_y$ ).

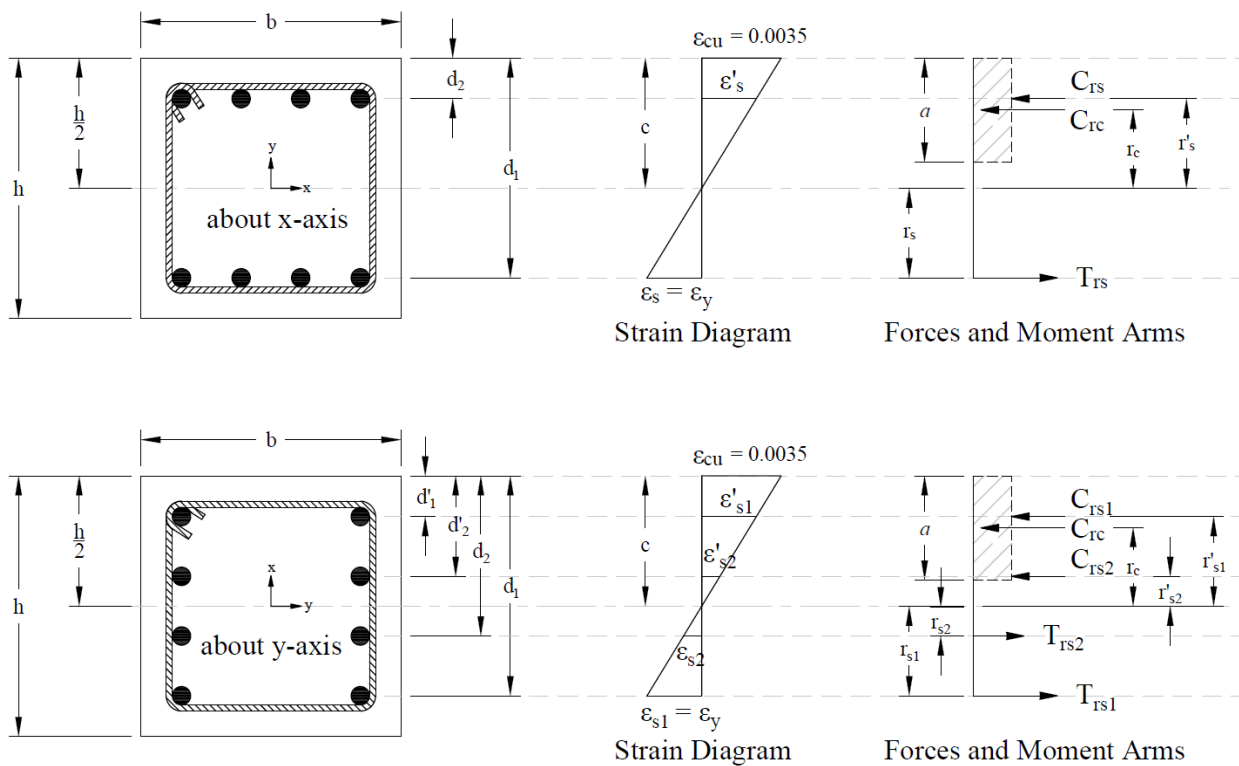


Figure 10 – Strains, Forces, and Moment Arms ( $f_s = f_y$  Moments About x- and y-axis)

When running about the Y-Axis, we have 2 bars in 4 layers instead of 4 bars in just 2 layers (about X-Axis) resulting in a completely different interaction diagram as shown in the following Figure.

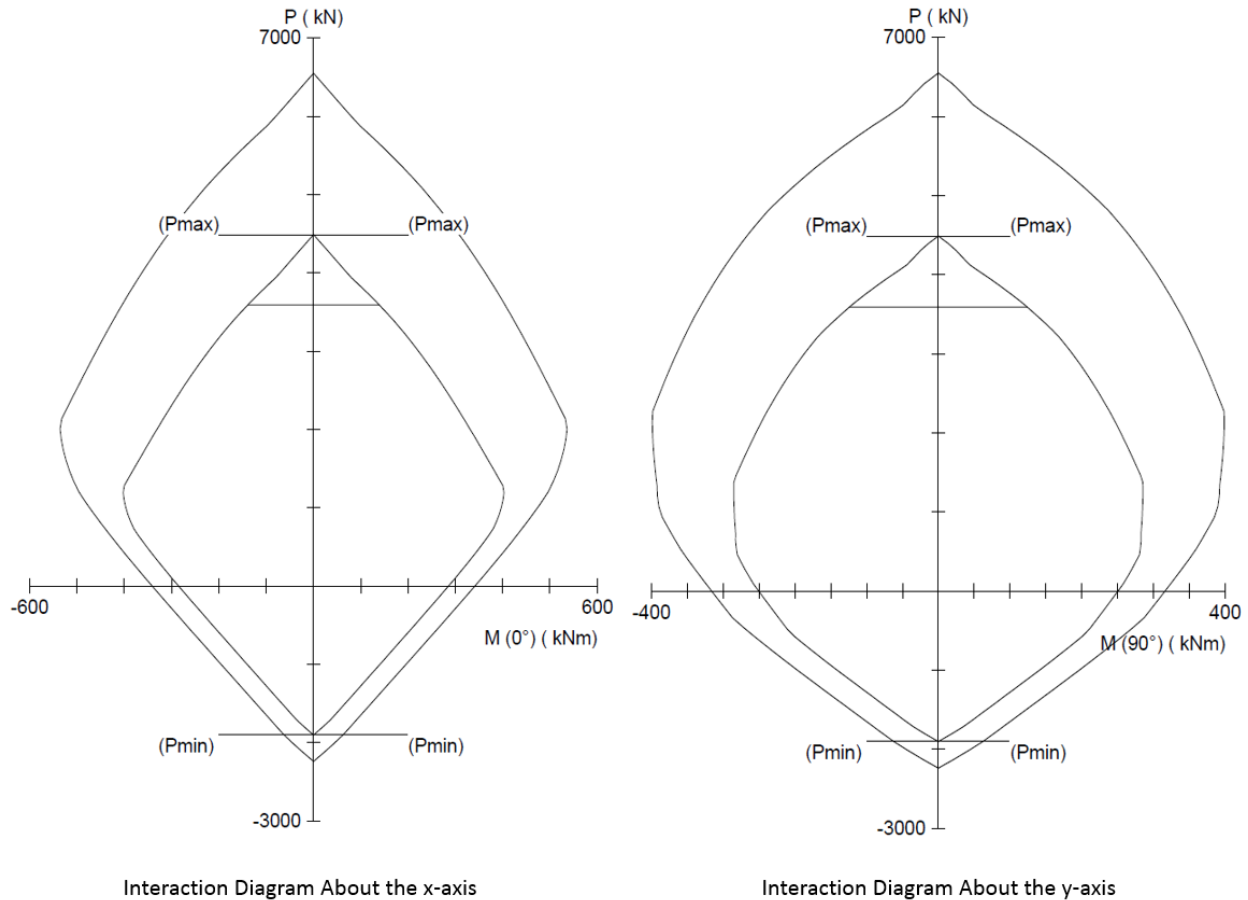


Figure 11 – Comparison of Column Interaction Diagrams about X-Axis and Y-Axis (spColumn)

Further differences in the interaction diagram in both directions can result if the column cross section geometry is irregular.

In most building design calculations, such as the examples shown for [flat plate](#) or [flat slab](#) concrete floor systems, all building columns are subjected to  $M_x$  and  $M_y$  due to lateral forces and unbalanced moments from both directions of analysis. This requires an evaluation of the column P-M interaction diagram in two directions simultaneously (biaxial bending).

StructurePoint's [spColumn](#) program can also evaluate column sections in biaxial mode to produce the results shown in the following Figure for the column section in this example.

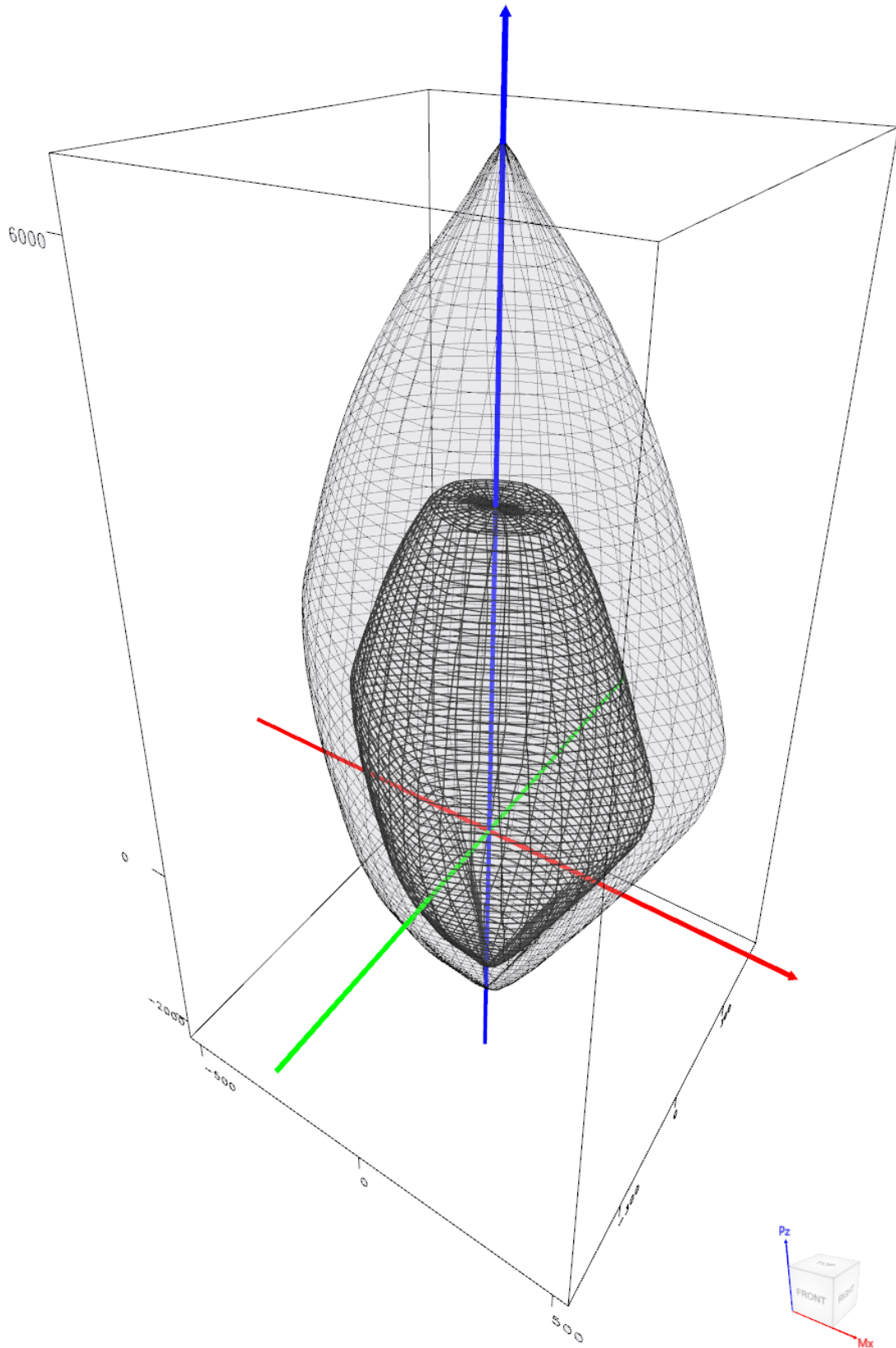


Figure 12 – Nominal & Design Interaction Diagram in Two Directions (Biaxial) (spColumn)